A keen observer of TFP and economic growth literature till date will most possibly come across two broad streams of study. While the first of set of research concerns theoretical analysis of TFP's impact on business cycles, cross-country growth differentiation, monetary policy and innovation strategies, the second strand of literature concerns its modelling and measurement with improved statistical methodology. For every observed economic event, there is a specific pattern and this is duly identified/modelled by state-of-the-art statistical tools. In our context, by testing for the persistence properties of TFP in a temporal setting, for instance, it is possible to know if a deterministic or stochastic component contributed to its growth over time. It is also possible to identify the type of economic growth system (i.e., exogenous or endogenous) that generated it. Since small and big changes in socio-economic structure and development are reflected in TFP, it is essential to choose an appropriate statistical test which best approximates its own 'evolutionary character' and at the same time characterizes the co-evolving pattern with other variables. Keeping up with this objective, this paper addresses the persistence test of TFP under classical and Bayesian paradigms and emphasizes that it is important to ask 'what is the probability that there is a unit root in TFP, then testing straightway if there is a unit root?' Nothing is certain in the phenomenal world. At best, one can say, an event is highly or remotely probable. The extent of probability has powerful implications for identifying socio-economic structure as opposed to the classical test of unit root where it can be generated by a host of events, viz., coordination failure of economic activities, existence of multiple equilibria and influence of nature, etc. In either case, the design of test matters for a thorough understanding of the socio-economy environment which generates it. This is discussed in Section 2 of the paper which provides an empirically testable model of TFP for classical and Bayesian unit root tests. Section 3 discusses data and empirical results and finally, concluding remarks are presented in Section 4.

2. Model of TFP persistence and testing via Bayesian route

Before we discuss the Bayesian unit root tests for persistence of TFP, let's define in the following the neoclassical production where output (Y) is produced by physical capital (K) and human capital (H) according to the Cobb-Douglas form:

$$Y_t = AK_t^{\alpha} H_t^{\beta} \quad (1)$$

Here α represents the share of capital in the production of one unit of output, *Y*. Human capital's share is presented by β. Various degrees of returns to scale occur when the combined value of α + β exceeds, is less than or equal to unity.

For instance, decreasing returns to scale to labour and capital occurs when the inputs' marginal productivities decline over time in the absence of any qualitative improvement of their efficiencies. Efficiency enhancement in inputs gives rise to increasing returns to scale. In the absence of *K* and *H*, output *Y* grows due to the *A*, the TFP or broadly due to innovation. Simple algebraic manipulation leads to the following TFP growth equation:

$$\frac{\dot{A}}{A} = \frac{\dot{Y}_t}{Y_t} - \alpha \frac{\dot{K}_t}{K_t} - \beta \frac{\dot{H}_t}{H_t} \qquad (2)$$

Let's denote by z_t , the TFP (i.e., $\dot{A}/A$) at time t . The evolutionary path of z_t is governed by how a shock imparted to z_t evolves over time. Denote by ε_t , a shock at time t , then negative shock (in terms of say, natural disaster, political turmoil, etc.) to TFP will result in decelerated growth and contractionary monetary policy, while a positive shock (in terms of innovation and diffusion, and good social development) will accelerate economic growth and motivate an expansionary monetary policy. Perpetuation of business cycles will be directly influenced by whether there is a persistent negative/positive TFP shocks. While it is required that at least a constant or increasing returns to scale should exist in the production technology to generate persistence in output (*Y*), such requirements are not necessary for enabling TFP persistence as this is determined outside the economic system.[1]

Statistically, the evolution of z_t can be represented by an autoregressive (AR) moving average (MA) specification. The endogenous or exogenous nature of TFP is characterized by whether z_t is a pure *AR* or a pure *MA* process. For our purpose, we assume that z_t is characterized by a history dependent property, such that the evolutionary path of z_t is given by the following *AR* (1) process without constant term,

$$z_t = \rho z_{t-1} + \varepsilon_t \text{ 3)} \qquad (3)$$

where it is further assumed that z_0 is a known constant, ε_t are *i.i.d* normally distributed with mean zero and unknown variance, σ^2, and $\rho \in S \cup \{1\}, S = \{\rho | -1 < l \le \rho < 1\}$, where l is the lower bound which determines the specification of the prior for ρ . We are interested in discriminating between a stationary model $(\rho < 1)$ and the nonstationary model with $\rho = 1$, i.e a random walk. The assumption of known z_0 points to the dependence of analysis on initial observations as the treatment of such condition will differ

---

[1] This is the case of exogenous TFP. Even if TFP is endogenous, such requirements are obsolete in this case as endogeneity of TFP depends on policies than on input use.

between stationary and non-stationary regions (Sims, 1988; Sims and Uhlig, 1991).

There is a sharp distinction between the testing procedure of existence of unit root between classical and Bayesian models.

While a knife-edge distinction is made between the presence and absence of a unit root in the form of testing whether $\rho = 1$ or $\rho < 1$ in classical Dickey-Fuller (1979) and its subsequent extensions, Bayesian mechanism asks how probable is the hypothesis that $\rho = 1$ against $\rho < 1$. This is because Bayesians are uncomfortable with testing a point hypothesis since it is not natural to compare an interval that receives a positive probability (the composite alternative $H_1 : \rho < 1$) with a point null hypothesis of zero mass (the null hypothesis $H_0 : \rho = 1$). They argue that the classical econometricians cannot give the probability that a hypothesis holds. What they can tell us is whether a hypothesis is rejected or not rejected (Koop, 1992). Moreover, classical test procedure is also criticized very strongly on the ground that it uses information that is not contained in the likelihood function which violates the likelihood principle[2] (Bauwens *et al.*, 1999). Sims (1988) argues the classical tests for unit root for their unusual nature of asymptotic theory leading to disconnected confidence intervals and the lack of power in small samples.

Equation 3 reflects that TFP at t, i.e. z_t depends on its past value as well as the stochastic error term, ε_t. History is shown to affect the evolution of z_t and as long as ε_t is an *iid* process, the evolutionary path of z_t will be solely determined by its past, z_{t-1} and the coefficient determining the extent of dependence is ρ. Question may arise then what is the probability that a particular value of ρ will occur given the value of z_t, i.e, one needs to find, $\Pr(\rho|z_t)$. This is arrived at by using Bayes theorem which amount to evaluating the product of the likelihood of $\Pr(z_t|\theta)$ and a prior probability $\rho(\theta)$, where $\theta = \{\rho, \sigma\}$. That is, the posterior information on ρ given the evolution pattern of z_t can be given by: $\Pr(\rho|z_t) \propto P(\theta).\Pr(z_t|\theta)$. Zellner (1971) proposes the following posterior odds ratio test to compare a sharp null hypothesis with a composite alternative hypothesis,

$$M_1 = M_0 \frac{\int_0^\infty p(\sigma) L(z \mid \rho = 1, \sigma, z_0) d\sigma}{\int_S \int_0^\infty p(\sigma) p(\rho) L(z \mid \rho, \sigma, z_0) d\sigma d\rho} = \frac{\Pr(\rho = 1 \mid Z)}{\Pr(\rho \in S \mid Z)} \quad (4)$$

where M_0 is the prior odds in favour of the hypothesis $\rho = 1$, M_1 is the posterior odds in favour of the hypothesis $\rho = 1$, $p(\rho)$ is the prior density of $\rho \in S$, $p(\sigma)$ is the prior density of σ, $L(z|.)$ is the likelihood function of

[2] This principle makes explicit the notion that only the observed data should be relevant to the inference about the parameter. This lies at the heart of the Bayesian inference.

the observed TFP data $z = (z_1,\dots z_T)'$, and finally, $z = (z_0, z')'$, all observed data. The posterior odds M_1 are equal to the prior odds M_0 times the Bayes factor. The Bayes factor is the ratio of the marginal posterior density of ρ under the null hypothesis $\rho = 1$ to a weighted average of the marginal posterior under the alternative using the prior density of ρ as a weight function. The specification of marginal prior of ρ and σ are assumed as: $\Pr(\rho = 1) = \omega = M_0/(1+M_0)$, $p(\rho|\rho \in S) = 1/(1-a)$, and $p(\sigma) \propto 1/\sigma$. The prior on ρ is uniform and has a discrete probability ω that $p = 1$. The prior on σ is diffuse, and corresponds to a uniform prior on $\ln\sigma$.

We can now provide the method to test for unit root using a uniform or flat prior (Sims, 1988). Suppose in our model we initially put probability α on the interval (0, 1), probability 1-α on ρ = 1, and independently a flat prior on $\ln\sigma^2$. The likelihood then has a normal inverse-gamma shape, conditional on the initial observations. The marginal likelihood for ρ is a *t*-distribution with $T - 1$ degrees of freedom and scale parameter.

$$\sigma_\rho = \sqrt{\left[\sigma^2 / \Sigma\, y(t-1)^2\right]}$$

This distribution is, for large *T*, very close to

$$N\left(\hat{\rho}, \sigma_\rho{}^2\right)$$

If we let Φ be the c.d.f. for the standard Normal distribution, ϕ be its p.d.f. and

$$\tau = (1-\hat{\rho})/\sigma_\rho$$

stand for the conventional *t*-statistic for ρ = 1, then for large *T* the odds ratio in favour of the ρ = 1 null hypothesis is 1-α times the normally shaped likelihood value at τ divided by α times the integral of the normally shaped likelihood over (0,1), i.e., the odds ratio in favour of ρ =1 null hypothesis is,

$$\frac{(1-\alpha)\phi(\tau)}{\sigma_\rho\{\alpha\Phi(\tau)\}}$$

assuming that the posterior probability on ρ <0 turns out to be negligible. This would reduce asymptotically to Schwarz criterion were it true that σ_ρ behaves asymptotically like a constant times $1/\sqrt{T}$.

Thus the criterion would be to compare:

τ^2(the square of conventional *t*-statistics) to 2 log(1- α)/α) - log(σ^2_ρ) + 2log(1-$2^{-1/s}$) $-2\log(\Phi(\tau))$ (where $\sigma^2_\rho = \sigma^2/\,\Sigma y(t-1)^2$, σ^2 is the variance of ε_t and for annual data s = 1) . The term $-2\log(\Phi(\tau))$ will be quite small when $\hat{\rho} < 1$ and is asymptotically negligible (Sims, 1988).

The test statistic is the square of the conventional t-statistic for ρ =1. This is compared with the Schwarz criterion, which has an asymptotic Bayesian justification and is considered as the asymptotic Bayesian critical value. Since the first and last terms in the expression for the critical value are constant for a given prior and data, a small τ favours no unit root. Therefore if t^2 is greater than the Schwarz limit, we reject the null hypothesis of a unit root.

Sims (1988) notes that it may not be reasonable to treat the prior as uniform over (0,1). Instead, we are interested in the case when the likelihood is concentrated somewhere near one. A lower limit for the stationary part of the prior is also specified such that the prior for ρ is flat on the interval (lower limit, 1.0). The concentration of the prior around 1 increases with the frequency of the data. If the prior is concentrated on (0.5, 1) for annual data, then for monthly data it is on (0.94, 1) where $0.94=0.5^{1/12}$. Following Sims (1988), $\alpha = 0.8$ is a reasonable choice since for this level the odds between stationarity and the presence of a unit root are approximately even.

3. Empirical analysis

3.1 Data and estimation issues

We estimated the posterior density of the autoregressive parameter in (3) and performed Bayesian unit root test for 23 African economies' TFP data for the period 1960-2003. Under non-stationary theory, estimation of posterior density is not straightforward as it involves lot of computational problems. Especially, it is required to solve a high-dimensional integral to integrate out the posterior function. Among several approaches to solve this problem, Simpson's integration rule is easy to use, at least when a flat prior is used for defining the posterior, which is the case with our specification. For details on Simpson's and other rules, the readers are referred to Bauwens *et al.* (1999).

Physical capital stocks were calculated according to the method used in Klenow and Rodriguez-Clare (1997). Initial capital stocks are calculated according to the formula:

$$\frac{K}{Y_{1960}} = \frac{I/Y}{\delta + \eta} \tag{5}$$

where (I/Y) is the average share of physical investment in output from 1960 through 2003, represents the average rate of growth of output per capita over that period, η represents the average rate of population growth over that period, and δ represents the rate of depreciation, which is set equal to 0.03. Given the initial capital stock, the capital stock of country i in period t is calculated by the perpetual inventory method:

$$K_{it} = \sum_{j=0}^{\infty}(1-\delta)^{t-j} I_{ij} (1-\delta)^t K_{1960} \quad (6)$$

TFP is then calculated as:

$$TFP_{it} = y_{it} - (1/3)k_{it} - (2/3)h_{it} \quad (7)$$

where the lower case letters for *K* and *H* represent *ln(K)* and *ln(H)*. The global share of labour and capital in the Cobb-Douglas production technology has been assumed to be approximately (1/3) and (2/3) respectively where a constant returns to scale is allowed in the aggregate growth of all inputs together. Data on physical capital stock is available with the authors which we do not present in the paper to save space. The real GDP per capita series, measured in thousand constant dollars in 2001 international prices, are extracted from the Penn World Table Version 6.1 (Summer and Heston, 2005), while the age-structured human capital data is sourced from IIASA-VID (see Lutz *et al.* 2007).

3.2 Bayesian test results

Table 1: Bayesian unit root test for Total Factor Productivity for Africa (1960-2003)
(*limit = 0.5, alpha = 0.8*)

Variables	**Squared t (t^2)**	**Schwarz limit**	**Marignal α**
Benin	2.304	4.772	0.406
Burkina Faso	13.240	5.056	0.003
Cambodia	1.454	5.697	0.624
Chad	3.064	5.637	0.419
Cote-I-vore	9.449	5.711	0.029
Egypt	0.034	7.851	0.908
Gambia	8.048	5.763	0.059
Ghana	8.603	5.047	0.032
Guinea	4.247	6.130	0.338
Kenya	6.119	4.834	0.094
Madagascar	0.304	6.544	0.818
Malawi	3.376	4.829	0.292
Mali	0.860	5.147	0.629
Mauritius	0.522	7.718	0.879
Morocco	31.199	6.813	0.000
Mozambique	2.219	5.240	0.474
Niger	0.498	6.404	0.792
Table 1 continued…			

Nigeria	3.900	5.554	0.313
South Africa	7.623	6.167	0.087
Togo	0.077	6.624	0.840
Uganda	0.892	5.776	0.696
Zambia	6.720	5.150	0.083
Zimbabwe	3.879	5.068	0.265
15 out 23 countries with t^2 < Schwarz limit			

Note Table 1:'*Alpha*' gives the prior probability on the stationary ρ; the remaining probability is concentrated on ρ = 1. The choice of the prior weight α can have a significant effect on the statistic given above. '*Marginal Alpha*' is the value for alpha at which the posterior odds for and against the unit root are even. A higher value of 'marginal alpha' favours the presence of unit root. Similarly, if t^2 is greater than Schwarz, we reject the null hypothesis of a unit root.

Before analysing the results of the posterior density, we first interpret the results of Bayesian unit root which utilizes the posterior odds ratio test (equation 4) with flat prior. From Table 1 it can be observed that t^2 < Schwarz (asymptotic Bayesian) limit for 15 out of 23 countries. That is, unit root cannot be rejected for fifteen countries, e.g. Benin, Cambodia, Egypt, etc., while for nine countries, e.g., Burkina Faso, Ghana, Kenya, Morocco and others, there is no unit root. If we examine the marginal α for each country's TFP, it provides evidence of the estimated probability of the existence of unit root persistence for respective countries. Among 23 countries, the marginal α is highest for Egypt (0.908) and lowest for Morocco (0.000). In other cases, when t^2 exceeds Schwarz limit, but have small values of marginal alpha (less than 0.5), it indicates that only a very strong prior on the unit root will overcome the data evidence against it.

3.3 Posterior analysis

As remarked before, the posterior value of the *AR* parameter (ρ) expected to be contaminated by non-stationarity conditional on the observed TFP data, z_t is estimated by employing Simpson's rule. The results of posterior ρ, its standard deviation and the corresponding range of integration are presented in Table 2. The range of integration is adjusted so as to achieve a normal distribution of ρ. The estimated values of ρ (column one) of Table 2 reflects the what we can expect about the possible non-stationary or stationary value of ρ conditional on the available set of information on total factor productivity data over four decades (1960-2003) for each country. Statistically, this is given by $\Pr(\rho|z_t)$. From Table 2, it is evident that the posterior value of ρ is greater than 0.5 for all countries under examination. This is highest for Burkina Faso (0.925) and Uganda (0.931) whereas for Kenya (0.514) and Mali (0.547), the posterior ρ is the lowest. The range of integration for all countries show that they swing widely between stationary and non-stationary regions. To summarize, the derived posterior values of ρ indicate in our case that the TFP series has high persistent character and that the history dependence feature reflected by ρ conditional on the initial and past information about the data is very high.

Table 2: Comparison of posterior mode for ρ Africa (1960-2003)

Countries (TFP)	ρ	σ_ρ	Range of Integration
Benin	0.680	0.139	0.20-1.20
Burkina Faso	0.925	0.140	0.40-1.50
Cambodia	0.826	0.058	0.60-1.10
Chad	0.813	0.088	0.50-1.10
Cote-I-vore	0.626	0.134	0.10-1.10
Egypt	0.784	0.137	0.30-1.30
Gambia	0.823	0.082	0.50-1.10
Ghana	0.685	0.147	0.15-1.20
Guinea	0.923	0.079	0.60-1.20
Kenya	0.514	0.137	0.05-1.00
Madagascar	0.646	0.147	0.15-1.15
Malawi	0.656	0.153	0.10-1.18
Mali	0.547	0.116	0.10-0.95
Mauritius	0.587	0.083	0.30-0.90
Morocco	0.572	0.134	0.10-1.05
Mozambique	0.785	0.130	0.30-1.30
Niger	0.572	0.182	0.00-1.20
Nigeria	0.847	0.096	0.50-1.20
South Africa	0.806	0.113	0.40-1.20
Togo	0.821	0.071	0.50-1.10
Uganda	0.931	0.067	0.70-1.20
Zambia	0.642	0.114	0.20-1.10
Zimbabwe	0.774	0.133	0.30-1.30

4. Conclusion and some economic interpretation

In this research note, we tested for the persistence of stochastic productivity shock in TFP series for a set of African countries for the period 1960-2003 using Bayesian framework. We found that the TFP series inherited high persistent character as reflected by derived posterior value of the autoregressive parameter. The results of Bayesian unit root test (using posterior odds ratio) also confirms the above conclusion. As such, the Bayesian test provided a realistic check of the probability of occurrence of TFP value in non-stationary region, which is in contrast to the classical test of unit root, a typical knife-edge test. High range of integration of posterior density in Table 2 indicates the nature of volatility of TFP for the examined period. It also reflects on the type of economic structure which is identified under the frequentist approach. With respect to economic policy, our finding of high probability of unit root in TFP calls for expansionary monetary policy as highly persistent TFP amounts to expansion of output in succeeding periods. Additionally, it could be argued that while the issue of the presence of an exact unit root in the classical sense fail to identify economic structure (Durlauf, 1989), Bayesian analysis could provide some intuition about the behaviour of the parameter and their relation with the structure of the economy.

5. References

Bauwens L., Lubrano M. and Richard J.-F. (1999), *Bayesian Inference in Dynamic Econometric Models*, Oxford University Press, Oxford.

Dickey D.A. and Fuller W.A. (1979), "Distribution of the Estimators for Autoregressive Time Series with a Unit Root", *Journal of the American Statistical Association*,74, pp. 427-431.

Durlauf S.N. (1989), "Output Persistence, Economic Structure and the Choice of Stabilization Policy", *Brookings Papers on Economic Activity*, 2, pp. 69-116.

Koop G. (1992), "Aggregate Shocks and Macroeconomic fluctuations: A Bayesian Approach", *Journal of Applied Econometrics*, 7, pp. 395-411.

Lutz W., Crespo-Cuaresma, J. and Sanderson W. (2008), "The Demography of Educational Attainment and Economic Growth", *Science*, 319, pp. 1047-1048.

Schotman P.C., van Dijk, H.K. (1991), "A Bayesian Analysis of the Unit Root Hypothesis in Real Exchange Rates", *Journal of Econometrics,* 49, pp. 195-238.

Sims C.A., Uhlig H. (1991), "Understanding Unit Rooters: A Helicopter Tour", *Econometrica*, 59, pp. 1591-1600.

Sims C.A. (1988), "Bayesian Scepticism on Unit Root Econometrics", *Journal of Economic Dynamics and Control*, 12, pp. 463-474.

Solow R.M. (1956), "A Contribution to the Theory of Economic Growth", *Quarterly Journal of Economics*, 70, pp. 65-94.

Zellner A. (1971), *An Introduction to Bayesian Inference in Econometrics*, John Wiley & Sons, New York.